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The Grube Idea of Teaching Arithmetic.

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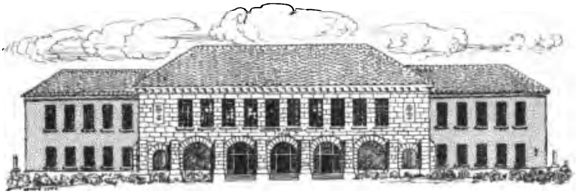

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ELEMENTS

OF THE

GRUBÉ METHOD.

CONTAINING THE LATEST IMPROVEMENTS MADE BY GRUBÉ'S FOLLOWERS IN GERMANY. REVISED, EXPLAINED, AND ILLUSTRATED FOR THE USE OF THE PRIMARY TEACHERS OF AMERICA.

LEVI SEELEY, M.A. PH.D. (LEIPZIG)
AUTHOR OF GRUBÉ'S ARITHMETIC.

DEPARTMENT OF
EDUCATION
OCT 2 5 1918
LELAND STANFORD
JUNIOR UNIVERSITY

"Bei deinem Thun gedenk' des Spruches ſtündlich:
Eins nach dem andern, aber — gründlich."

NEW YORK AND CHICAGO:
E. L. KELLOGG & CO.

646602

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1890.

INTRODUCTION.

IN 1842 Grube issued his work entitled "Leitfaden für das Rechnen in der Elementarschule," (Guide for Reckoning in the Primary School). This work introduced a new idea in teaching number; it carries forward the four processes,—addition, subtraction, multiplication and division at the same time. The effect of this was to revolutionize the method of teaching number in Germany; the methods now employed there are founded upon Grube's idea. Undoubtedly the inspiration of Grube's work was found in Pestalozzi, whose teachings deeply stirred Germany just before Grube came on the stage. Pestalozzi had learned from Rousseau that the proper teacher of children is Nature: that her laws and her methods are the only safe and sure guide. Grube made a practical use of it in his system of teaching number. He holds that the carrying of the four processes together, the mastering of each number as a step by itself, the use of objects leading from the concrete *thing* to the abstract *number*, is the order of nature. His method starts out with processes which are based entirely upon observation, but gradually objects are abandoned and a greater use made of the reasoning powers.

Grube's system is planned to cover the first four years of school, and contemplates an equivalent of three hours per week for four years, during which the fundamental rules, compound numbers, and fractions are mastered. As a great many children do not remain in school longer than four years, and as the subjects mastered are nearly all that are of value in practical life, the Grube system is peculiarly suited to the American schools. If this has been thoroughly taught, a supplementary course of two years should complete the ordinary practical arithmetic.

There has been a very earnest inquiry among teachers in America for methods of teaching number founded on pedagogical principles. Every teacher of young children should be acquainted with the Grube system, for a faithful application of it will save a vast amount of the time of the children and of the labor of the teacher ; but more important still, a comprehension of the real significance of number is acquired. Primary teaching is always the most difficult and requires the most skilful teachers, because the child is not able to direct his energies and must be led by the teacher.

The design of this book is to make the method of Grube so clear that any teacher can easily apply it. To teachers in cities, where a course in number is specifically marked out, this little book will be found most suggestive and helpful.

It is proposed in this book simply to present the *Elements* of the Grube system ; to such as desire to understand it more fully, the larger book, "Grube's Method of Teaching Arithmetic," published by E. L. Kellogg & Co., which gives the system complete, is recommended.

The plan of the work is as follows :

Part I, Numbers from 1 to 100........	First year, 1 to 10. Second year, 10 to 100.
Part II, All numbers above 100...........	Third year.
Part III, Fractions......................	Fourth year.

LEVI SEELEY.

LAKE FOREST, ILL., 1890.

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DIRECTIONS TO TEACHERS.

THE first course includes whole numbers 1 to 100. Two years are required for this work, the first year being spent on 1 to 10 and the second on the rest, 10 to 100.

2. The number-lesson must at the same time be a language-lesson. The child should give his answers in complete sentences, plainly spoken, with clear accent. So long as the child is not master of the language necessary to express the operations performed with the number, he is not master of the representation or idea of the number itself; he does not know the number.

3. An example is not finished when the answer is found, but when it has been analyzed. The explanation may be taken as a test that a pupil has completely mastered the example.

4. The pupil must be led to speak independently and not to depend upon half the answer being put into his mouth by the teacher. Answers in concert may interchange with individual answers in order that the interest of the class be maintained.

5. The uniform objects to be used are the fingers and blocks; for blackboard or slate use simple straight lines. Too many kinds of objects must not be used. The child has only a certain amount of strength and mental power which he can apply, and his interest must not be divided. The mental comprehension of number is disturbed if things which awaken other ideas or desires are employed. The mind is capable of only a certain amount of interest, and when this interest is wholly or partly withdrawn but little can be expected

for the particular thing at hand. For this reason, while teaching the abstract number, there should be but few things shown the child, and these should be simple and uniformly the same. The best things are blocks, which awaken little interest in themselves, and these must be the chief objects used throughout. Other objects should be referred to after the child has a number well fixed, by way of application, but should seldom be shown him, at least during the study of the first numbers. Thus apples, nuts, etc., which awaken desire, stimulate the appetite, and thus divide the attention, must not be used as objects in teaching number. All the interest which the child gives to the color, taste, etc., of objects is just so much lost to number.

6. The operations in any step consist simply of comparing and measuring what has been gained in the preceding steps with that which is new. The child proceeds from the known to the unknown, from the easy to the difficult; hence the method must follow this psychological law. The *pure* number is first learned, and then it is applied to *things* in order to fix it and make its practical use apparent.

7. The work of teaching a number is not complete until the child has been taught to make neatly and with despatch the figure which stands for the number. This makes excellent employment for the children at their seats, and is a good preparation for written arithmetic, for which the foundation is being laid.

8. The teacher must go slowly—he must not measure the ability of the child by his ability; he must bring himself down to the level of the child's mind; be patient; repeat everything many times; review daily; use many examples and lead the children to make original problems; do not expect too much of children; give them the kind and quantity of food that they can digest, always remembering that real sound, intellectual growth is slow, especially at the beginning.

THE FIRST YEAR: 1-10.

THE work of the year embraces a study of the numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Each number is taken by itself, measured by those that precede it, compared and studied in all its possible operations.

FIRST STEP.

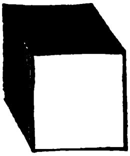
THE ONE.

The one can only be measured by itself. The child has only to learn the idea of unity.

NOTE.—It may be said that the child already knows the *one*, and that it is folly to spend time in teaching it. But it must not be forgotten that the child when he enters school must begin the study of number, must begin to form habits of correct speaking and thinking, must learn to observe carefully, to tell what he sees, and to answer in complete sentences.

Then, too, the Grube system builds step by step, always making use of the knowledge possessed. The *one* is the first step For these reasons and for these purposes a short time can be profitably spent with the *one*. It may be further added that the fact of the child's knowing the *one* will make it especially valuable in getting him to feel confidence in himself as he uses the knowledge that he possesses, an important matter for the child when he begins school.

I. The Pure (Abstract) Number.



(*The teacher shows the block.*) How many have I?
P. You have *one*.
T. Charles, take this block. Now how many has Charles?
P. Charles has *one*.

T. Charles, give it to me. Now how many has Charles?
P. Charles has ***none.***
T. How many have I?
P. You have one.

The teacher must lead the children to watch his movements, and to describe accurately what he does.

Many other questions similar to the above should be given. Great care must be taken with the language. Correct statements must be given. ***Never accept anything else.***

NOTE.—The idea of a number is given the child by the use of an object or of objects which are placed before him to see. He sees the concrete form, but does not name it. It suggests to him the abstract number. Thus when the child sees one block he is led to think of it as a *one*, two blocks as a *two*, three blocks as a *three*, etc. Do not let the child say "one block," "two blocks," but simply "one," "two," etc. Requiring the child to name the object withdraws his attention from the number itself. It is much easier for him to speak of the abstract number, and it is also easier for him to read and write it. The block gives him the picture of the number, and that is all that is wanted of it; after getting the picture of it he only needs to use the name that stands for the number. After becoming thoroughly familiar with the number, the child must go a step farther and use it with various objects and make application of it. This procedure is entirely psychological in that it proceeds from the simple to the complex, from the known to the unknown.

T. Now let us put this ***one*** on the board (***makes a drawing of the block***). How many have I?
P. You have one.
T. But it takes too long to make this kind of a one; we will let this [I] stand for one.* How many have I?
P. you have I.

The use of the mark is simply to save time. The child readily proceeds from the object [the block] to its picture, and from that to the straight mark [I] as standing for one.

T. Now we will learn to make something else that stands for one. Watch me and see what it is. It is this *1*. What does that stand for?
P. That stands for one.
T. Now you make that on your slates.

By this method the pupils are taught the figure. The block, its picture, and the mark [I] must be taught as the number *one.* The child thinks of them only as the *one.* But he must learn to make the character which stands for the one; namely, the figure [1]. No number must be left and no step be considered complete until the pupils have learned to make the figure which stands for the number. Great care must be taken by the teacher not to confuse the ***number*** with the ***figure.***

---

* The teacher should not use the parenthesis.

II. The Applied Number.

T. What thing do you find but once in the school-room?
P. I find *one* stove (desk, clock, etc.).
T. What have you *one* of at home?
P. I have one dog (cat, sled, etc.).

But little time need be given to the one, the aim being chiefly to get good expressions from the children, and to start them in the mode of thought to be pursued hereafter with the other numbers.

SECOND STEP.

THE TWO.

I. The Pure Number.—*Measuring and Comparing.*

T. How many have I?
P. You have *two.*
T. (*Takes a block in each hand and holds up one.*) What have I here?
P. You have *one.*
T. And what have I here? (*the other hand.*)
P. You have also *one.*
T. Now watch and see what I do. (*Moves the hands holding the blocks slowly together.*) What did I do?
P. You put one and one together.

Continue this operation until they can make a good statement of the fact.

T. Very well. Now we will put this little story on the board.

□ and □ make □ □. That is good; but I can make these marks [| |] instead of the pictures [□ □], and that will be easier. What have I here [| |]?
P. You have two.
T. But I can make something else which means two. It is this: *2*. What have I now?
P. You have two.

The teacher should in this manner teach each figure. When the pupils have learned the number, and have seen it expressed by pictures of blocks and by marks, they are then ready to learn the simplest way of expressing it, that is, by the figure. Make the figure slowly before the class, so that they can imitate your manner of making it.

T. Now what have I? (Two blocks shown together.)
P. You have a *two*.
T. How many 2's have I?
P. You have one 2.
T. How many times have I 2?
P. You have 2 one time or once.
T. How many does one 2 make?
P. One 2 makes 2.
T. Now let me write—One 2 is 2.

This will be reached only after repeated and patient efforts, but it pays to give the necessary time, because it is fundamental and all later work will be better and more easily done. Show the blocks and lead the child to see for himself. Always go back to the blocks when the child is in doubt. After the child has learned the figure it may be used to indicate the number. Until then use objects, pictures, or marks to stand for the number.

T. Again, notice what I have. (*The two blocks in one hand.*)
P. You have 2.
T. Now see what I do. (*The teacher slowly takes* 1 *from* 2)
P. You took 1 away from 2.
T. And what does that leave?
P. It leaves 1.
T. Who will tell me the whole story about what I did?
P. You took 1 away from 2 and it left 1.
T. Good; we will write that also.

□ □ less □ leaves □.

Very well; now we will write that another way.

2 less 1 leaves 1.

Let us now see what else we can do. (*The teacher holds the* 2 *blocks before the children and takes* 1 *away.*) What have I done?

P. You have taken 1 away.

(*Teacher then takes the other away.*) Now what have I done?

P. You have taken 1 away again.

T. How many times have I taken 1 away from 2?

P. You have taken 1 away two times.

T. Then how many 1's are there in 2?

P. There are two 1's in 2.

T. Now we will write that on the board.

In 2 there are two 1's.

Or we may say 2 divided by 1 makes 2.

This last statement will be somewhat difficult for the child, and should not be attempted until by the method above given he has mastered the idea. Then we may teach this as another way of expressing the same thing. It is necessary that the child learn this way of expressing division, as it is the simplest expression and is the form that he will most often meet with. After mastering the idea it will not take long for the child to learn the form in common use.

T. Now we will write what we have learned about 2.

1 and 1 make 2.
One 2 is a 2 or makes a 2.
Two 1's are a 2 or make a 2.
2 less 1 is 1.
In a 2 there are two 1's.*
In a 2 there is one 2.

At this time the signs can be taught. Write each expression using the words, and underneath write the same expression using the signs. For example:

1 and 1 make 2.
1 + 1 = 2.
2 divided by 1 makes 2.
2 ÷ 1 = 2.

Require the child to read the expression containing the words, and then that containing the signs. He must read both exactly alike. Hereafter use only signs, and require the pupils to make and use them. Go slowly, repeat many times, seek to get correct expressions, teach the children to observe accurately what you do and to describe it.

---

* I can take 1 from 2 twice.

The four processes must be repeated until the pupils can give all operations with great rapidity. They must also be able to make their statements fluently, to read readily from the board, and to write exercises from dictation, all to be included within the 2.

The pupil must be able to answer such combinations as follows, the teacher developing them by use of the blocks:

What number is found twice in 2?

Of what number is 2 the double?

Of what number is 1 the half?

What number must I double in order to get 2?

I know a number which has 1 more than 1. What is it?

What number must I add to 1 in order to get 2?

All possible combinations of the 2 should thus be given.

II. The Applied Number.

The. pupils are now prepared to apply their knowledge in practical examples embracing other objects than the blocks. It is no longer necessary to show them the objects. Let them also make examples.

John had 2 cents and spent 1 cent for candy. How much had he left?

A lead pencil cost one cent. How much will 2 pencils cost?

Fred had 1 dime in his savings bank; Mary had twice as many. How many dimes had Mary?

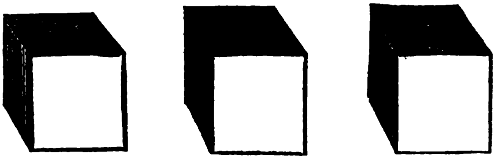
If a cake cost 1 cent, how many cakes can you buy for 2 cents?

Charles had 2 apples and Frank had half as many. How many had Frank?

George had 1 apple and John twice as many. How many had John?

THIRD STEP.

THE THREE.



I. The Pure Number.—*Measuring and Comparing.*

(a)—With one.

The cubes should be placed on a table where all the class can see what is done with them. It is inconvenient to operate with three or more blocks in the hands. Therefore, the blocks must be shown on the table.

T. How many have I? (*Shows the* 3 *blocks separately.*)
P. You have 1 and 1 and 1.
T. How many is that?
P. 1 and 1 and 1 = | | |.
T. Good; I will print this little story on the board:

□ + □ + □ = □□□.

then

1 + 1 + 1 = | | |.

When sure that the pupils know the 3, he writes 1 + 1 + 1 = 3, teaching the figure 3 by the same method as the 2 was taught. The symbol representing the number must be taught as soon as the pupils have obtained the idea of the number, and not before. After they have learned the figure which stands for a number, no other characters need be used to represent the number.

As the pupil knows the signs +, —, =, etc., they may be freely used instead of and, less, equals or makes, etc.

T. Now tell me how many times I have 1. (*Picks up the blocks one after another, the children counting.*)
P You have one 3 times.
T. One 3 times makes how many?
P. One 3 times makes 3.

T. Then how many are 3 times one?
P. 3 times 1 are 3, or

$$3 \times 1 = 3.$$

T. (*Holds up the* 3 *blocks and takes* 1 *away.*) Now what have I done?
P. You have taken 1 away from 3.
T. How many are left?
P. There are 2 left.
T. Who will write that for me on the board?
P. (*Child writes* 3 − 1 = 2.)
T. (*Takes another block away from the remaining* 2.) What else have I done?
P. You have taken 1 away from the 2.
T. How many does that leave?
P. It leaves 1.
T. Now read 3 — 1 — 1 = 1.
T. Still, again, how many times can I take 1 away from 3? (*Takes* 1 *away* 3 *times, children counting.*)
P. You can take 1 away 3 times.
T. Then how many ones are there in 3?
P. There are 3 ones in 3.
T. Then I will write 1 in 3 = 3 or 3 ÷ 1 = 3. Now see what we have learned.

$$\begin{matrix}1\\1\\1\end{matrix}\begin{cases}1+1+1=3\\3\times 1=3\\3-1-1=1\\3\div 1=3\end{cases}$$

(b)—*Measuring with* 2.

In the same manner the following tables will be developed, the children always seeing the manipulations with the blocks and acquiring the statements of themselves. If a child hesitates and does not yet comprehend, go over the operations again. Call him up and let him handle the blocks until he has mastered the process and understands the relations. Let him do it himself as much as possible.

$$\begin{matrix}1 & 1\\ & 1\end{matrix}\begin{cases}2+1=3,\ 1+2=3\\1\times 2+1=3\\3-2=1,\ 3-1=2\\3\div 2=1,\text{ and 1 remainder.*}\end{cases}$$

---

* This should be developed as follows:
T. How many have I here? (*Holding up the* 3 *blocks.*)
P. You have 3.
T. How many ones in it?
P. There are 3 ones.

Give questions like the following in every possible form and repeating in many ways:

3 is 1 more than what?
3 is 2 more than what?
2 is 1 less than what?
In the same manner find that—
1 is 2 less than 3.
3 is 3×1.
1 is $\frac{1}{3}$ of 3.[*]
What 3 equal numbers make 3?
What numbers with 1 make 3?
The 3 contains a 2 and a 1.

Use only the blocks, the fingers and marks. Do not divide the attention by the introduction of new objects or things that awaken a desire of possession in the child. Require the pupils to make the figures and count 1, 2, 3 forwards and backwards, thus 1, 2, 3, 3, 2, 1.

Rapid Work.

Teacher gives examples orally as rapidly as possible, the children giving only the answer.

How many are $3 - 1 - 1 + 1$?
$3 \times 1 - 2 \times 1 - 1 = ?$
$1 + 1 \times 1 + 1 - 3 = ?$
$3 - 2 + 1 + 1 - 2 = ?$
$2 \times 1 + 1 - 2 + 1 - 1 = ?$

Many examples of this kind should be given until the pupils are able to give the answer instantly. This is largely oral work.

T. Now see how many twos you can find. (*Children take the blocks and find out for themselves*) How many are there?
P. There is one 2.
T. Who will find another 2?
P. There is no other 2, there is only 1 left.
T. Then how many twos in 3?
P. There is one 2.
T. And what is there left?
P. There is 1 left.
T. Now tell me the whole story.
P. In 3 there is one 2 and 1 left.
T. Very well, but I like the word *remainder* instead of *left*. Now try again.
P. In 3 there is one 2 and 1 remainder.

[*] To teach this I would take 3 blocks and place 1 block near them. Then ask which is the larger and how many times. Reverse the process, and ask which is the smaller, how many of the ones it takes to make a three, and finally, what part of the three the one is. It will not be at all difficult for the child to use and understand such small fractions.

Combining.

From what number can you take the double of 1 and still have 1 remaining?

What number is 3 times 1?

I use a number once, and then once again, and then once again, and obtain 3. What is the number?

II. The Applied Number.

Anna had a 3-cent piece, and bought 2 cents' worth of candy. How much change should she get?

If a pencil cost 1 cent, how much will 3 pencils cost?

Charles has 2 apples and 1 apple. How many apples has he?

Mary has 3 roses, which she divides among friends, giving each one. How many friends receive roses?

Martha, Fanny, and William have each 1 book. How many books have they?

A boy had 3 apples and ate 1 apple. How many had he left?

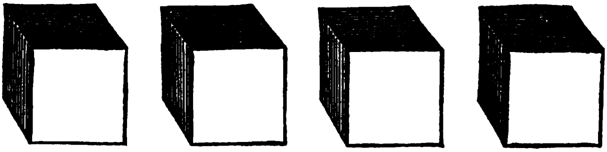
Henry has 2 dollars. How much more must he earn in order to have 3 dollars?

An uncle divided 3 dimes equally among his 3 nieces. How much did each receive?

My father gave each of his boys 1 dollar, and it took 3 dollars. How many boys had he?

FOURTH STEP.

THE FOUR.



It will not be necessary longer to pursue the question and answer method, as it has been sufficiently illustrated. The course indicated should be pursued for all subsequent numbers. Develop all the relations which come under, "Measuring and Comparing" by use of the blocks. Lead the children to the statement of what you develop. The further you proceed the greater opportunity for variety, but limit that to such as may be obtained by use of such objects as have been heretofore specified, namely, blocks, fingers, marks.

I. The Pure Number.—*Measuring and Comparing.*

(*a*) With 1.

1 1 1 1 — 4
| *1* — $1 + 1 + 1 + 1 = 4$, $1 + 1 = 2 + 2 = 4$.
| *1* — $4 \times 1 = 4$.
| *1* — $4 - 1 - 1 - 1 = 1$.
| *1* — $4 \div 1 = 4$.

(*b*) With 2.

$2 + 2 = 4$.
| | *2* — $2 \times 2 = 4$.
| | *2* — $4 - 2 = 2$.
$4 \div 2 = 2$.

(*c*) With 3.

$3 + 1 = 4$, $1 + 3 = 4$.
| | | *3* — $1 \times 3 + 1 = 4$.
| — $4 - 3 = 1$, $4 - 1 = 3$.
$4 \div 3 = 1$ and 1 remainder (3 in 4 once, and 1 remainder.)

How many legs has a lion?
How many wheels has a bicycle?
4 is how many more than 3?

Bring out the following facts with the blocks:

4 is 1 more than 3, 2 more than 2, 3 more than 1.
3 is 1 less than 4, 1 more than 2, 2 more than 1.
2 is 2 less than 4, 1 less than 3, 1 more than 1.
1 is 3 less than 4, 2 less than 3, 1 less than 2.
4 is 4 times 1, 2 times (or double) 2.
1 is one fourth of 4, 2 is one half of 4.
Of what equal and what unequal numbers is 4 made up?

Rapid Work.[*]

$2 + 2 - 2 + 1 \times 1 + 1 - 2$ doubled $= 2$
$4 - 1 - 1 - 1 + 1 - 3$ is how much less than 4?
$3 - 1 + 2 - 3 + 2 + 1 + 2 = ?$
$2 \times 2 \times 1 - 3 + 2 - 1 \times 2 + 3 = ?$
$1 + 1 + 1 - 2 \times 4 - 3 \times 2 = ?$
$4 + 2 + 1 + 1 - 3 - 1 = ?$

Combining.

What number must I take 2 times in order to get 4?
Of what number is 4 the double?
One is one fourth of what number?
Two is one half of what number?
Of what number is 1 the fourth?
What number can be taken twice away from 4?
How much must I add to the half of 4 in order to get 4?
How many times 1 is the half of 4 less than 3?
If I take 1 from 4, how many times 1 have I left?
How much is the half of 4 more than the third of 3?
How much is the fourth of 4 less than the half of 4?

II. The Applied Number.

How many cents in 2 two-cent pieces?

How many lemons can you buy for 4 cents if each costs 2 cents?

If a top costs 2 cents, how much will 2 tops cost?

One quart has 2 pints. How many pints in 2 quarts?

William had 4 peaches and ate 2. How many had he left?

Anna received an orange on Monday, one on Tuesday, one on Wednesday and one on Thursday. How many did she receive in all?

Give many examples from the life of the children until they can make application of all the relations contained within the 4. Do not let them leave a number for a new step until they can make the figure, perform rapidly all the combinations whether given to them orally, or written on the board, make complete statements of such combinations, and apply them in their own surroundings. The great success of this method depends upon the thoroughness in these particulars.

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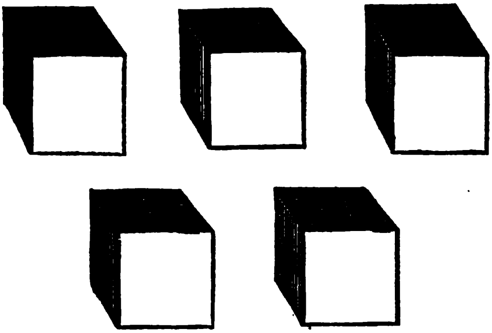
[*] These expressions are intended only for dictation, the pupil working as fast as they are dictated, and obtaining the result of each step with no reference to what is to follow. Thus $3 - 1 \times 2 - 3 + 2 + 1 + 2$ would be when worked out from dictation $3 - 1 = 2$, $2 \times 2 = 4$, $4 - 3 = 1$, $1 + 2 = 3$, $3 + 1 = 4$, $4 + 2 = 2$. Of course the pupils do not repeat the numbers as in the exercises here given, but obtain each result mentally as soon as the teacher dictates. The end sought is *rapidity* as well as accuracy. When expressions are written for the pupils, care must be taken to have them mathematically correct and not to include combinations beyond the 4, or the number which is being taught. The above expressions, as well as those which follow later under the head of "Rapid Work," may not always be correct when taken as a whole and considered as a mathematical expression; but they are correct as dictation exercises, and a few are given for the purpose of suggesting the method to the teacher. Such exercises will be found very valuable.

FIFTH STEP.

THE FIVE.



As it is difficult to work with five blocks in the hands, they should be manipulated on a table before the children so that they can be plainly seen. It would be better still if the children could be gathered around a large table and each have the same number of blocks, so as to perform the same work as the teacher does with hers. The blocks can very well be dropped and only the fingers used after this.

I. The Pure Number.—*Measuring.*

(*a*) With 1.

$$1\ 1\ 1\ 1\ 1 \quad 5$$

$$\begin{array}{ll} | & \mathit{1} \\ | & \mathit{1} \\ | & \mathit{1} \\ | & \mathit{1} \\ | & \mathit{1} \end{array} \left\{ \begin{array}{l} 1 + 1 + 1 + 1 + 1 = 5 \\ 5 \times 1 = 5,\ 1 \times 5 = 5 \\ 5 - 1 - 1 - 1 - 1 = 1 \\ 5 + 1 = 5 \\ \end{array} \right.$$

(*b*) **With 2.**

$$\begin{array}{rr} |\,| & 2 \\ |\,| & 2 \\ | & 1 \end{array} \left\{\begin{array}{l} 2+2+1=5 \\ 2\times 2+1=5 \\ 5-2-2=1 \\ 5\div 2=2\ (1) \end{array}\right.$$

(*c*) **With 3.**

$$\begin{array}{rr} |\,|\,| & 3 \\ |\,| & 2 \end{array} \left\{\begin{array}{l} 3+2=5,\ 2+3=5 \\ 1\times 3+2=5 \\ 5-3=2,\ 5-2=3 \\ 5\div 3=1\ (2) \end{array}\right.$$

(*d*) **With 4.**

$$\begin{array}{rr} |\,|\,|\,| & 4 \\ | & 1 \end{array} \left\{\begin{array}{l} 4+1=5,\ 1+4=5 \\ 1\times 4+1=5 \\ 5-4=1,\ 5-1=4 \\ 5\div 4=1\ (1) \end{array}\right.$$

After the pupils are familiar with all the combinations, they may be required to fill out missing numbers; for example:

3 + ... = 5, 4 × 1 + ... = 5, 4 × .... + 1 = 5.

5 is 1 more than 4, 2 more than 3, 3 more than 2, 4 more than 1.

4 is 1 less than 5, 1 more than 3, 2 more than 2, 1 more than 3.

3 is 2 less than 5, 1 more than 2, 2 more than 1.

2 is 3 less than 5, 1 more than 1.

$5 = 5 \times 1$.

$1 = \frac{1}{5} \times 5$ (1 is the fifth part of 5.)

The five consists of unlike numbers, 3 + 2, and of 2 like numbers and 1 unlike number, 2 + 2 + 1.

Rapid Work.

5 − 2 − 2 + 2 − 1 × 2, the half, less 1 = ?

2 × 2 + 1 − 3 × 1 × 2 − 3 − 4 = ?

Combining.

How much must I add to 3 in order to get 5?

How much must I take away from 5 in order to get 2?

What number is the fifth part of 5?

How many times 2 have I added to 1 to get 5?

I have taken from a number twice 2 and have 1 left. What is the number?

If to 2 times 2 I add 1, what do I get?

I take 3 times 1 from a number and have 2 left. What is the number?

What number shall I add to 2 to get 5?

If I add 1 to a number I get 5. What is the number?

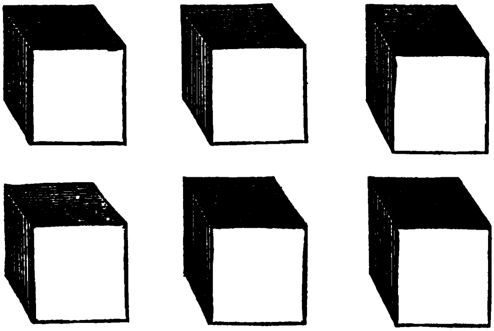
What must I add to the half of 4 to get 5?
Take 2 times 1, then add it to 2; this lacks how many of 5?
4 times one third of 3 added to 1 makes what?
Take 3 from 5 and how many will 2 times this lack of 5?

II.—The Applied Number.

THE teacher will be able to make examples applying the five without further illustration. The pupils also should be encouraged to make examples.

SIXTH STEP.

THE SIX.



The pupil is now able to fill out the operations himself according to the method already pursued. If he can do that readily, it may not be necessary to use the objects longer, except when the child is puzzled. Many German teachers use no objects in teaching number after the four. Just as soon as the child is familiar with the method, and can grasp the idea of the number without the visible object before him, the objects should be abandoned. But blocks should always be at hand to be used in removing doubt in the child's mind, when a point is not perfectly clear to him. Let objects be the means to an end, and when the end is attained, abandon their use.

I. The Pure Number.—*Measuring.*

$$\begin{array}{rr} \mathrm{I} & 1 \\ \mathrm{I} & 1 \\ \mathrm{I} & 1 \\ \mathrm{I} & 1 \\ \mathrm{I} & 1 \\ \mathrm{I} & 1 \end{array} \left\{ \begin{array}{l} 1+1+1+1+1+1=6 \\ 6\times 1=6,\ 1\times 6=6 \\ 6-1-1-1-1-1=1 \\ 6\div 1=6 \end{array}\right.$$

Teach each figure as soon as the child has use for it.

$$\begin{array}{rr} \mathrm{II} & 2 \\ \mathrm{II} & 2 \\ \mathrm{II} & 2 \end{array} \left\{ \begin{array}{l} 2+2+2=6.\quad (2+2=4,\ 4+2=6) \\ 3\times 2=6 \\ 6-2-2=2 \\ 6\div 2=3 \end{array}\right.$$

$$\begin{array}{rr} \mathrm{III} & 3 \\ \mathrm{III} & 3 \end{array} \left\{ \begin{array}{l} 3+3=6 \\ 2\times 3=6 \\ 6-3=3 \\ 6\div 3=2 \end{array}\right.$$

$$\begin{array}{rr} \mathrm{IIII} & 4 \\ \mathrm{II} & 2 \end{array} \left\{ \begin{array}{l} 4+2=6,\ 2+4=6 \\ 1\times 4+2=6 \\ 6-4=2 \\ 6\div 4=1\ (2) \end{array}\right.$$

$$\begin{array}{rr} \mathrm{IIIII} & 5 \\ \mathrm{I} & 1 \end{array} \left\{ \begin{array}{l} 5+1=6,\ 1+5=6 \\ 1\times 5+1=6 \\ 6-5=1 \\ 6\div 5=1\ (1) \end{array}\right.$$

$6=5+1,\ 4+2,\ 3+3,\ 2+4,\ 1+5$
$5=6-1,\ 4+1,\ 3+2,\ 2+3,\ 1+4$
$4=6-2,\ 5-1,\ 3+1,\ 2+2,\ 1+3$
$3=6-3,\ 5-2,\ 4-1,\ 2+1,\ 1+2$
$2=6-4,\ 5-3,\ 4-2,\ 3-1,\ 1+1$
$1=6-5,\ 5-4,\ 4-3,\ 3-2,\ 2-1$
$6=6\times 1,\ 3\times 2,\ 2\times 3$
$3=\frac{1}{2}\times 6$ (is half of 6)
$2=\frac{1}{3}\times 6$
$1=\frac{1}{6}\times 6.$

Of what 3 like numbers is 6 composed? Of what 3 unlike?

The following tables can be profitably given:

(*a*) $1+5=6$
$2+4=6$
$3+3=6$
$4+2=6$
$5+1=6$

(*b*) $6-1=5$
$6-2=4$
$6-3=3$
$6-4=2$
$6-5=1$
$6-6=0$

(*c*) $1 + 2 = 3$
$3 + 2 = 5$
$5 + 1 = 6$

(*d*) $5 - 2 = 3$
$3 - 2 = 1$

Odd numbers : 1, 3, 5 ; 5, 3, 1
Even numbers : 2, 4, 6 ; 6, 4, 2.

$2 + 2 = 4$
$4 + 2 = 6$

$6 - 2 = 4$
$4 - 2 = 2$
$2 - 2 = 0$

Many other tables can be made embracing multiplication and division as far as the 6. The pupils can easily be led to make these themselves. Dictate numbers, and require the pupils to name or write them promptly, so as to test their knowledge of the order of the numbers, and their ability to make the figures. For example: Write the numbers from 1 to 6. What number comes after 4? What comes before 4? What comes between 2 and 4? Express the number that comes after 5. Express the number between 3 and 5.

Rapid Work.

$1 \times 2 + 1 \times 2 - 1 \times 1 - 5 + 5 = ?$
$4 + 2 - 3$ is how much less than 6?
$3 - 2 \times 5 + 1 - 4 \times 2 \div 2 + 4 - 5 = ?$
$5 - 4 + 3 \div 2 + 1 \times 2 \div 3 = ?$

Allow no counting of the fingers or use of objects in this operation. The pupils must *know* every operation instantly, and be able to perform rapidly and accurately without any hesitation. This is a test of the thoroughness of the work.

Combining.

What number can you take 3 times from 6 and twice from 4?

How many times 1 has half of 6 more than half of 4, and how much less than 5?

I have taken a number twice away from 6 and have 2 left. What is the number?

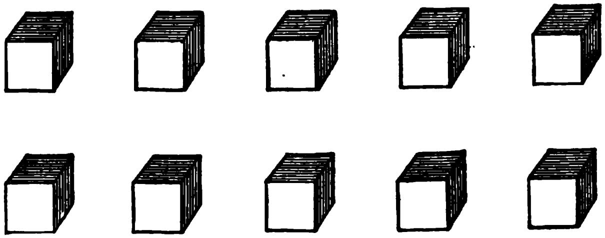
How many times is ½ of 6 contained in 4? The half of 4 = what part of 6? What number is 3 times 2?

II. The Applied Number.

It will not be necessary to illustrate the Seventh, Eighth and Ninth Steps,* the same plan being followed as in the preceding Steps. The teacher can easily make the tables, and the pupils even will be found able to do so. Require them to do all they can for themselves. Never leave a number for a new one until the former is mastered. Gradually the tables of compound numbers will be learned. They should be introduced as soon as a number is reached which admits of any part of a table being taught. Bring each step of knowledge into practical use as soon as possible. Make examples from the duties of home life. All the steps—*measuring*, *rapid work*, *combining*, and *applying* must be given with each number.

TENTH STEP.

THE TEN.



We have now reached the first number which must be considered as another kind of unity, or another kind of One,—the Ten. So we write again the figure 1, but to show that this 1 contains ten times as much as the first 1, we move it one place to the left, and say, this 1 is a ten. The vacant place of the simple 1 will be indicated with the cipher, so—10. The pupils should be taught as follows:

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* "Grube's Method of Teaching Arithmetic" has all of these Steps, published by E. L. Kellogg & Co.

Show me 10 fingers. Now 1 finger. Indicate the 1 finger with a figure. Indicate the fingers of both hands with a figure.

The children are shown how they must write 1 in the ten's place, and the cipher at the right for unit's place.

Ten splints also may be used, and then bound into a bundle to represent 1 ten. This must be continued until the pupils comprehend the 10 and its relation to the unit.

Measuring.

At this point Grube abandons the writing out of the tables, such as 10 with 1, 10 with 2, 10 with 3, etc., as practised with all preceeding numbers. The teacher can require the pupils to do so if he deems it desirable; but it will probably be found unnecessary to go farther in this direction.

The 10 consists of two equal numbers, 5+5; of 5 equal numbers, 2+2+2+2+2; of 2 equal and 1 unequal numbers, 3+3+1; of 4 unequal numbers, 1+2+3+4.

1 is the half of 2
the third of 3
the fourth of 4, etc.
2 is the half of 4
the third of 6
the fourth of 8, etc.
10 is 10 times 1
5 times 2
2 times 5
3 is the half of 6
the third of 9
4 is the half of 8
5 is the half of 10
6 is 6 times 1

6 is 3 times 2
2 times 3
9 is 9 times 1
3 times 3
8 is 8 times 1
4 times 2
2 times 4
7 is 7 times 1
5 is 5 times 1
4 is 4 times 1
2 times 2
3 is 3 times 1
2 is 2 times 1
1 is 1 times 1

What numbers go without remainder into 10, 9, 8, 6, 4?
What are only divided by 1 and themselves?

Rapid Work.

Same as before.

Combining.

What number has 1 more than the double of 3?

How much is 2×5 greater than the difference between 3×3 and the double of 4?

A father divided 10 apples among his 4 children so that each older received 1 more than the next younger. How many did each receive?

The 10 consists of the 4 unequal numbers, 1+2+3+4; each is 1 larger than the next below. Therefor the father could give the youngest 1 apple, the next 2, etc.

Herman said, "I am 5 times as old as my brother." The brother was 2 years old. How old was Herman?

II.—The Applied Number.

In 10 pints how many quarts?

In 10 days how many weeks and days?

In 10 cents how many 2-cent pieces? 3-cent pieces? 5-cent pieces?

Fred had 6 cents, 3 cents, and 1 cent. He went to a stationer and bought 4 sheets of paper at 2 cents a sheet, and two sheets at 1 cent a sheet. Did he have money enough?

Karl had the same amount of money and bought 3 sheets at 3 cents each. How much did he have left?

How many pints of milk can be bought for 10 cents if one pint costs 2 cents? 5 cents?

A dime has 10 cents. How many 2-cent pieces equal a dime? How many 5-cent pieces?

This completes the first school year, and the most important steps in number have been mastered. One year is not too long if the work has been thoroughly done. Of course the child knows only the numbers 1 to 10; but he *knows* them thoroughly and can use his knowledge; they are of real value to him.

At the 10, if not before, the use of objects should be abandoned entirely. The child is now able to gain the abstract idea without the help of objects. Objects become a cumbrance as soon as the child can do without them. Some think they can be abandoned after the 4 or 5. Each teacher must settle the question with the class he may have. When the class are able to get the idea without the objects, then is the time to give them up. Certainly that end will be reached when the 10 is completed.

Reviews must be frequent, and every step must be mastered before proceeding to the next. After knowledge has been obtained by illustration and observation it must be thoroughly memorized.

THE SECOND YEAR: 10-100.

OBSERVATIONS.

1. Grube says: "Fingers and lines continue to be used for illustration. One can well say that Nature has given to man the decimal system of number in the hand."

2. The procedure in the following steps is the same as that given for the smaller numbers. Multiplication and division should be given both as written and oral work, while addition and subtraction need only be oral. The pupils must continue the "measuring" of each new number by the numbers from 1 to 10 until the greatest mechanical skill is reached. This mechanical skill is connected with the greatest self-activity on the part of the pupil.

3. For the operations with the pure as well as the applied numbers, a greater diversity in the manner of expression in the examples can be employed, in order that the pupil may become more and more free from the formulas of the earlier work. Applied examples should be gathered from the pupil's surroundings, from material with which he is familiar.

ELEVENTH STEP.

THE ELEVEN.

I.—The Pure Number.

10 times one or 10 ones make 1 ten.

If I have 10 ones taken together, I have 1 ten and no (0) ones or units besides.

| | | | | | | | | | = 1 ten and 0 ones = 10.

If another one is added, it belongs to the second ten.

| | | | | | | | | | 10 + 1 = 11.
|

What is the one at the right? What the 1 at the left? Where does the one (unit) belong? How many ones must be added in order to make the second ten full? What do we call 1 ten and 1 one in one word? What is the 11?

Oral.

Measure with 1.

1 + 1 + 1 + etc. = 11. (1 + 1 = 2, 2 + 1 = 3, 3 + 1 = 4, etc.)
11 × 1 = 11.
11 − 1 − 1 − 1, etc. = 1. (11 − 1 = 10, 10 − 1 = 9, 9 − 1 = 8, etc.)
11 ÷ 1 = 11.

Measure with 2.

2 + 2 + 2 + 2 + 2 + 1 = 11.
5 × 2 + 1 = 11.
11 − 2 − 2 − 2 − 2 − 2 = 1.
11 ÷ 2 = 5 (1).

Measure with 10.

10 + 1 = 11.
1 × 10 + 1 = 11 (1 ten + 1 one = 11).
11 − 10 = 1.
11 ÷ 10 = 1 (1) (In 11 is 1 ten + 1 one).

Each pupil gets by this means a principle, and as he knows the course to follow, assistance from the teacher should cease.

All numbers hereafter are measured only by the numbers from 1 to 10.

Written.

|||||||||| = 10 + 1 = 11.
|

11 = 11 × 1
5 × 2 + 1
3 × 3 + 2
2 × 4 + 3 etc.

11 ÷ 1 = 11
2 = 5 (1)
3 = 3 (2)
4 = 2 (3) etc.

Comparison (oral).

11 = 10 + 1, 9 + 2, 8 + 3, etc.
11 = 11 × 1, 1 = $\frac{1}{11}$ × 11 (1 is $\frac{1}{11}$ of 11).
Form 11 from 3 equal numbers and 1 unequal number.
4 equal and 2 unequal numbers.
5 equal numbers and 1 unequal number.
4 unequal numbers.

Rapid Work.

I have 6 cents, 3 cents, 1 cent, and 1 cent, and give away 4 cents, 2 cents, and 3 cents. How much have I left?

Let all the possible combinations within the 11 be given orally until the pupil can reckon as rapidly as the teacher can give them.

Combining.

How many 1's must I add to 5×2 to get 11?
From what number must I take 3×3 to get 2?
How often can I take the fourth part of 8 away from 11?
What number is 1 ten greater than 1?
What is the difference between 4×2 and 11?

II.—The Applied Number.

Give practical examples as before.

TWELFTH STEP.

THE TWELVE.

I. The Pure Number.

| | | | | | | | | | = 10 + 2 = 12
| |

Oral.

$1 + 1 + 1 + 1$, etc., $= 12$
$12 \times 1 = 12$
$12 - 1 - 1 - 1$, etc., $= 1$
$12 \div 1 = 12$

$2 + 2 + 2 + 2 + 2 + 2 = 12$
$6 \times 2 = 12$
$12 - 2 - 2 - 2 - 2 - 2 = 2$
$12 \div 2 = 6$

$3 + 3 + 3 + 3 = 12$
$4 \times 3 = 12$
$12 - 3 - 3 - 3 = 3$
$12 \div 3 = 4$

Measure in the same manner by all the numbers as far as 10.

Written.

12 = 12 × 1
6 × 2
4 × 3
3 × 4
2 × 5 + 2
etc.

12 ÷ 1 = 12
2 = 6
3 = 4
4 × 3
5 = 2 (2)
etc.

12 = 11 + 1, 10 + 2, 9 + 3, etc.
12 is 1 more than 11, 2 more than 10, etc.
1 is the twelfth part of 12.
2 is the sixth part of 12.
3 is the fourth part of 12.
4 is the third part of 12.
6 is half of 12.

Rapid work, combining, and applying will follow as before.

The teacher will be able to supply the other Steps up to twenty,* following the same general plan as that given with eleven and twelve. All possible applications of compound numbers should be made so that the child will know nearly all of the tables before he gets to the twenty.

TWENTIETH STEP.

THE TWENTY.

I.

| | | | | | | | | | 10 + 10 = 20.
| | | | | | | | | |

Now how many tens have we?

Show me 20 fingers. (Let two children hold up both hands twice.)

Here are twenty sticks. How many bundles of tens can we make from them? We bind them together and have what number?

Write 20 so that the lines fall in twos.

|| || || || ||
|| || || || ||

How many fours are there?

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* They are given in "Grube's Method or Teaching Arithmetic."

The 10 can be separated in two equal parts; how many the 20?

$$\left.\begin{array}{ccccc}\bullet&\bullet&\bullet&\bullet&\bullet\\ \bullet&\bullet&\bullet&\bullet&\bullet\\ \bullet&\bullet&\bullet&\bullet&\bullet\\ \bullet&\bullet&\bullet&\bullet&\bullet\end{array}\right\}4 \times 5 = 20$$

How many points in one of these horizontal lines? In one of the perpendicular lines?

Now write 4 dots in a horizontal line; how many rows are there?

$$\left.\begin{array}{cccc}\bullet&\bullet&\bullet&\bullet\\ \bullet&\bullet&\bullet&\bullet\\ \bullet&\bullet&\bullet&\bullet\\ \bullet&\bullet&\bullet&\bullet\\ \bullet&\bullet&\bullet&\bullet\end{array}\right\}5 \times 4 = 20$$

Of what equal numbers does 20 consist?

$$20 \times 1,\ 10 \times 2,\ 5 \times 4,\ 4 \times 5,\ 2 \times 10.$$

Of what number is 20 the double? The fourfold? The fivefold? The tenfold?

What part of 20 is 2, 4, 5, 10?

Measure with 1.

1, 2, 3, 4, 5, 6, (In which ten are we? How many does it lack of being full?) 7, 8, 9, 10, 11, 12, 13, 14, 15.

How many have we in the second ten? How many in all? How many ones must we take to fill the second ten?

$$20 \times 1 = 20.$$

20, 19, 18, 17, (How many ones must we still take away before we get to the first ten?) etc.

$$20 \div 1 = 20.$$

I can take 1 away from 20 twenty times, or 1 is contained in 20 twenty times.

Measure with 2.

2, 4, 6, 8, 10, 12, (How many times 2 have we now?) 14, 16, (How many now?) 18, 20.

$$10 \times 2 = 20.$$

20, 18, 16, 14, etc.

$$20 \div 2 = 10.$$

Measure with 3.

3, 6, 9, 12, 15, 18, (How many must we still add to get 20?) 20.

$$6 \times 3 + 2 = 20.$$

How many times can I take 3 from 20?

As often as I take 3 away, we will count a finger.

20, 17, 14, 11, 8, 5, 2.

$$20 \div 3 = 6\ (2), \text{ etc.}$$

$$\begin{array}{rlcrl} 20 = & 20 \times 1 & \qquad & 20 \div & 1 = 20 \\ & 10 \times 2 & & & 2 = 10 \\ & 6 \times 3\ (2) & & & 3 = 6\ (2) \\ & 5 \times 4 & & & 4 = 5 \\ & \text{etc.} & & & \text{etc.} \end{array}$$

Rapid Work and Combining as before.

II.—The Applied Number.

20 things = a score.

20 cents = 2 dimes.

How many weeks in 20 days?

How many pounds in 20 ounces?

Charles had 20 cents to spend. He bought 4 apples at 2 cents each, a ball at 6 cents, gave his sister 2 cents, and kept the balance. How much did he keep?

James is 20 years old and his brother Chester is 5 years younger than he, while his cousin John is 3 years younger than Chester. How old is John?

2 dimes is how many times 5 cents?

20 cents equal how many 2-cent pieces?

A farmer had 20 pecks of corn. How many bushels had he?

A board is 20 inches long. How much of it must I saw off in order to leave just a foot?

A merchant found just 20 feet in a piece of cloth. How many yards did the piece contain?

A milkman had 2 ten-gallon cans full of milk. He sold 11 gallons in the morning, and the rest in the evening. How much did he sell in the evening?

THIRTIETH STEP.

THE THIRTY.

I.—The Pure Number.

| | | | | | | | | | 10 + 10 + 10 = 30.
| | | | | | | | | |
| | | | | | | | | |

(3 times the fingers of both hands.)

If I add 1 to the 29 the third ten will be full.
3 tens taken together is called 30.

Measure with 1.

Count by ones upwards: 1, 2, 3, 4, 5, 6, 7, (Stop! a pupil says: "We are in the first ten; it lacks 3 of being full, and 20 more of 30.") 8, 9, 10, 11, 12, 13, (We are in the second ten, etc.) 14, 15, etc.

30 × 1 = 30.

Count downwards: 30, 29, 28, (We are in the third ten; from the third ten 2 have been taken away.) 27, 26, etc.

30 + 1 = 30.

Measure with 2.

2, 4, 6, 8, 10, 12, 14, (We are in the second ten; it lacks 3 twos of being full, and 5 twos more of completing the 3 tens or thirty.) etc.

15 × 2 = 30.

Count downwards with 2: 30, 28, 26, 24, (We are in the third ten, have taken 3 twos from the third ten and there remain 2 twos in the third ten).

30 + 2 = 15.

Continue with the other numbers in the same way.

Measure with 10.

10 + 10 = 20, 20 + 10 = 30.
3 × 10 = 30
30 — 10 = 20, 20 — 10 = 10.
30 + 10 = 3.

$$\left.\begin{array}{cccccccccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{array}\right\} 3 \times 10 = 30.$$

30 = 30 × 1
15 × 2
10 × 3
7 × 4 (2)
etc.

30 ÷ 1 = 30
2 = 15
3 = 10
4 = 7 (2)
etc.

Rapid work the same as before.

Combining.

30 — 19.
19 = 10 + 9, 30 — 10 = 20, 20 — 9 = 11, 30 — 19 = 11.

How do you get the double of 15?

15 = 1 ten and 5 units; 2 × 1 ten = 2 tens; 2 × 5 units = 10 units = 1 ten; 2 tens + 1 ten = 3 tens = 30.

Compare 30 with 16.

30 = 3 tens; 16 = 1 ten + 6 units. I must add to the 6 units 4 units to complete the second ten, and still 1 ten to get 3 tens. Therefore 30 has 1 ten + 4 units = 14 more than 16, and 16 is 1 ten + 4 units, or 14 less than 30.*

10 × 3 = 6 times what number?
If I take 3 × 5 from a number, I get 5 × 3 as remainder. What is the number?

II.—The Applied Number.

The teacher will easily supply the examples.†
All the numbers to 100 will be treated in the same way.

Written.

Pupils should write the above tables as follows:

1 + 2 = 3 or 1 + 7 = 8 or 100 — 8 = 92
3 + 2 = 5 8 + 7 = 15 92 — 8 = 84
5 + 2 = 7 15 + 7 = 22 84 — 8 = 76
7 + 2 = 9 22 + 7 = 29 etc.
etc. etc.

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* This method of adding and subtracting is universally practised at present in the German schools, with most excellent results. The tens are the stepping-stones of the method.

† See the larger work for more examples.

$100 = 100 \times 1$
50×2
33×3 (1)
25×4
etc.

$100 \div 1 = 100$
$2 = 50$
$3 = 33$ (1)
$4 = 25$
etc.

$100 = 99 + 1, 98 + 2, 97 + 3$, etc.

100 is 100-fold of 1
50-fold of 2
25-fold of 4
20-fold of 5
5-fold of 20
2-fold of 50

1 is $\frac{1}{100}$ of 100
2 is $\frac{1}{50}$
4 is $\frac{1}{25}$
5 is $\frac{1}{20}$
20 is $\frac{1}{5}$
50 is $\frac{1}{2}$

HUNDREDTH STEP.

(*The Last Step of the First Course.*)

THE HUNDRED.

The counting upwards and downwards with numbers from 1 to 10, beginning with both 1 and 2, must be done without hesitation, rapidly and accurately. In counting in concert, the teacher should frequently stop the pupils with questions as before. To illustrate:

Counting with 2, beginning with 1: 1, 3, 5, 7, 9, 11, 13, (Stop! In what ten are we? How many units does it lack of being complete? How many tens still remain to complete 100? How many units?) 15, 17, 19, 21, etc.

Counting with 6, beginning with 2: 2, 8, 14, 20, etc.

Counting with 9, beginning with 4: 4, 13, 22, etc.

Counting with 6, beginning with 99: 99, 93, 87, 81, etc.

This method of counting is of great value, and should be practised until the child is able readily to answer questions like the above.

Rapid Work.

$100 \div 2 + 10 \div 30 \times 25 \times 2$?
$4 \times 25 - 50 \div 5 \times 10 - 75 \times 3 + 25$?

All possible combinations in multiplication have already been learned in the course of the various exercises, but may now be arranged in the multiplication table and practised.

2×2=4								
2×3=6	3×3=9							
2×4=8	3×4=12	4×4=16						
2×5=10	3×5=15	4×5=20	5×5=25					
2×6=12	3×6=18	4×6=24	5×6=30	6×6=36				
2×7=14	3×7=21	4×7=28	5×7=35	6×7=42	7×7=49			
2×8=16	3×8=24	4×8=32	5×8=40	6×8=48	7×8=56	8×8=64		
2×9=18	3×9=27	4×9=36	5×9=45	6×9=54	7×9=63	8×9=72	9×9=81	
2×10=20	3×10=30	4×10=40	5×10=50	6×10=60	7×10=70	8×10=80	9×10=90	10×10=100

Besides what has already been given, the four fundamental rules must be practised with numbers above 10 until the pupils are brought to complete mastery of them, and are able to make various combinations with great rapidity and accuracy. A few examples follow which must be worked only orally:

a. (Addition)—

$$42 + 15 + 26 + 37$$

etc.

The method of working is as follows: $42 + 10 = 52$, $52 + 5 = 57$, $57 + 20 = 77$, $77 + 6 = 83$, $83 + 30 = 113$, $113 + 7 = 120$.*

b. (Subtraction)—

$$97 - 12 - 34 - 16 - 27$$

etc.

To be worked as follows: $97 - 10 = 87$, $87 - 2 = 85$, $85 - 30 = 55$, $55 - 4 = 51$, $51 - 10 = 41$, $41 - 6 = 35$, $35 - 20 = 15$, $15 - 7 = 8$.

c. (Multiplication)—

$$17 \times 9$$
$$15 \times 37$$

Employ the following method of solution: (17×9) $10 \times 9 = 90$, $7 \times 9 = 63$; $90 + 60 = 150$, $150 + 3 = 153$. (Likewise 15×37) $10 \times 37 = 370$, $5 \times 30 = 150$, $5 \times 7 = 35$; $370 + 100 = 470$, $470 + 50 = 520$, $520 + 35 = 555$.

d. (Division)—

$$72 \div 3$$
$$84 \div 12$$

Method as follows: $72 \div 3$ is $60 \div 3 = 20$ and $12 \div 3 = 4$. This $72\ [60+12] \div 3 = 24\ [20+4]$.

Another: $84 \div 12$ is $60 \div 12 = 5$ and $24 \div 12 = 2$. Thus $84\ [60+24] \div 12 = 7\ [5+2]$.

Combining.

The teacher will write a row of figures on the board to be added, subtracted, multiplied, and divided, placing the signs between. He then points to the exercises, and the pupils solve them rapidly. The teacher must always solve the examples himself. This holds good for all work in arithmetic, and is very important.

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* This method is based on the importance of the tens as the basis of all computations, and it will be found that pupils learn to operate with great rapidity and accuracy.

Such examples as the following must be rapidly worked by the pupils:

$$(3 \times 29) - (4 \times 16) + 7 \div 10 \times 9 \times 3?$$
$$(15 \times 6) - (45 \div 9) \times 37?$$

As a test whether every member is comprehended by the pupil, let him name the factors of numbers from 1 to 100 without naming the numbers. Prime numbers can only be measured with 1. For example: 1 × 1, 2 × 1, 3 × 1, 2 × 2, 5 × 1, 2 × 3, 7 × 1, 2 × 4, or 2 × 2 × 2, 3 × 3, etc.

It is important that the children know readily that 52 = 4 × 13, 68 = 4 × 17, 95 = 5 × 19, and so on to 100.

Such exercises as the following should be thoroughly drilled:

2 × 11 = 22	2 × 12 = 24	2 × 13 = 26
3 × 11 = 33	3 × 12 = 36	3 × 13 = 39
etc.	etc.	etc.

The prime and composite numbers can be arranged as follows:

1 prime number
2 " "
3 " "
4 = 2 × 2
5 prime number
6 = 2 × 3
7 prime number
8 = 2 × 2 × 2
9 = 3 × 3

and so forth till 100.

Examples should be made by both teacher and pupils until the pupils are able to thoroughly apply the number 100.

This completes the work of the first two years according to Grube. Later educators in Germany accept the first year's work, but modify the second very materially.*

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* The complete Grube gives a valuable appendix to the second year's work.

NUMBERS—ABOVE 100.

THE THIRD YEAR: FIRST HALF.

1. As the numbers between 100 and 1000 are combinations of the numbers within the first hundred, the only purpose of this course is to reduce them to their elements.

2. Thereby the pupil comes into possession of the secret of all accurate and rapid mental work in arithmetic, namely, always to operate with the smallest possible numbers; hence he needs none of the so-called "arithmetical knack."

3. In order to lead to an allsided representation of the number, it is impossible to consider all of the fundamental rules at once as heretofore; this will receive wider attention in the second half of the year. Mental and written arithmetic are now united at every step.

4. As the necessity of isolating each number now disappears, and because the allsided penetration and comprehension of each number must take place, the material will be divided in two parts only:

A. The pure number: measuring, separating, comparing, and combining.

B. The applied Number.

The child is now sufficiently mature gradually to leave simple mechanical processes and make more use of the understanding and reason. He must, however, master the processes, so as to be able to give them rapidly and almost mechanically.

A.—Allsided Contemplation of the Pure Number.

(First Quarter.)

FIRST STEP.

Measuring of the numbers by the units of the Decimal System, by units, tens, and hundreds.

a. (Oral.)

Count upwards and downwards from 100 to 1000. 10 splints can be bound together, and that is 1 ten. Around 10 of these bundles (100) a wide ribbon may be tied; 10 of these bundles make 1000. In this way the pupil gains a comprehension of 1000.

During the counting the pupils must often be stopped and questioned as to which hundred and which ten they are in : How many units and tens are lacking in the tens and hundreds respectively, and in the hundreds how many are lacking from a thousand?

For example : The teacher gives 768. The pupil will explain :

768 = 7 hundreds, 6 tens, 8 units. It lacks 2 units of completing the 7th ten, then 3 tens of completing the 8th hundred, and finally 2 hundreds of completing 1000.

What number is composed of 4 hundreds, 6 tens, and 7 units?

How many units in 5 hundreds, 9 tens, and 9 units?

How many units in one thousand? How many tens? Hundreds?

Of what does 669 consist?

$$(6 \times 100) + (6 \times 10) + (9 \times 1)$$

b. (Written.)

To make it easier for beginners in writing, use the following plan:

h.	t.	u.
1	0	1 = 101
4	8	0 = 480
10	0	0 = 1000

Numbers must be dictated for the pupils to write. When written on the blackboard, the figures should be named and the numbers read, etc., in order to acquire perfect mastery of the subject.

Finally, analogous to the oral work, dictated numbers should be written out as follows:

$$\begin{aligned} 615 &= 6 \times 100 + 1 \times 10 + 5 \times 1 \\ 204 &= 2 \times 100 + 0 \times 10 + 4 \times 1 \\ 390 &= 3 \times 100 + 9 \times 10 + 0 \times 1 \\ 1000 &= 10 \times 100 + 0 \times 10 + 0 \times 1 \end{aligned}$$

or

$$\begin{aligned} 615 &= 600 + 10 + 5 \\ 204 &= 200 + 4 \\ 390 &= 300 + 90 \\ 1000 &= 1000 \end{aligned}$$

In the following steps we shall give only one form, which will answer for both oral and written work. Neither is to be omitted, but they must be united, the oral taking precedence in order of time. Follow the plan suggested in the First Step.

SECOND STEP.

The pure hundreds measured with hundreds.

Measuring, comparing, rapid work, combining, are to be the same as in the First Course. With the number 2 in the First Course we obtained the following scheme:

$$\begin{aligned} 1 + 1 &= 2 \\ 2 \times 1 &= 2 \\ 2 - 1 &= 1 \\ 2 \div 1 &= 2 \end{aligned}$$

Conformably to this the pupil now learns by easy transition:

200.

$$\begin{aligned} 100 + 100 &= 200 \\ 2 \times 100 &= 200 \\ 200 - 100 &= 100 \\ 200 \div 100 &= 2 \end{aligned}$$

What number is contained twice in 200?
Of what number is 200 the double?
Of what number is 100 the half?
What number must I double in order to get 200?

Give many other examples, following the method employed in teaching the 2. See Second Step in the First Course.

In the same way take 300, 400, etc.

THIRD STEP.

Mixed hundreds measured with mixed hundreds.

220 = 2 × 110, also 1 × 220
440 = 4 × 110, also 2 × 220
660 = 6 × 110, also 3 × 220
880 = 8 × 110, also 4 × 220, 2 × 440
990 = 9 × 110, also 3 × 330

How may 888, 999, be considered?

888 = 8 × 111, also 4 × 222, 2 × 444
999 = 9 × 111, also 3 × 333

999 + 333 =	888 + 222 =
999 + 3 =	888 + 4 =
999 + 111 =	888 + 8 =
etc.	etc.

Of what number is 110 the ¼, ⅛, ⅑?
What is ¼ of 488?
What number can I take 4 times from 488?
What number is contained 4 times in 844?
½ of 844 is how much greater than ¼?
⅓ of 333 is ⅑ of what number?
Of what number is ⅓ of 333 the ninth? (⅓ of 333 = 111. 111 is ⅑ of 9 times 111 = 999.)
Compare 365 with 244.

365 = 3 hundreds + 6 tens + 5 units;
244 = 2 hundreds + 4 tens + 4 units.

3 hundreds − 2 hundreds = 1 hundred; 6 tens − 4 tens = 2 tens; 5 units − 4 units = 1 unit. Therefore 365 − 244 = 1 h. + 2 t. + 1 u. = 121, or 365 is 121 greater than 244, and 244 is 121 less than 365.

What is the difference between 743 and 120?

743 = 7 h. + 4 t. + 3 u.; 120 = 1 h. + 2 t. + 0 u.; 7 h. − 1 h. = 6 h. 4 t. − 2 t. = 2 t.; 3 u. − 0 u. = 3 u. Therefore 743 − 120 = 6 h. + 2 t. + 3 u. = 623.

FOURTH STEP.

Measuring of hundreds with tens.

a. The pure hundreds.

Since 100 = 10 × 10,

2 × 100 or 200 = 2 × 10 × 10 = 20 × 10
3 × 100 or 300 = 3 × 10 × 10 = 30 × 10
10 × 100 or 1000 = 10 × 10 × 10 = 100 × 10 = 1000.

b. **Hundreds with tens.**

Since $100 = 10 \times 10$,

$$\begin{aligned} 110 &= (10 \times 10) + (1 \times 10) = 11 \times 10 \\ 120 &= (10 \times 10) + (2 \times 10) = 12 \times 10 \\ 130 &= (10 \times 10) + (3 \times 10) = 13 \times 10 \end{aligned}$$

c. **Hundreds with tens.**

Since $100 = 10 \times 10$,

$$\begin{aligned} 101 &= (10 \times 10) + 1 \\ 109 &= (10 \times 10) + 9 \\ 906 &= (90 \times 10) + 6 \\ 814 &= (81 \times 10) + 4 \end{aligned}$$

How many tens in 500?
What number $=$ 53 tens?
What number $=$ 9 units more than 53 tens?

FIFTH STEP.

Measuring a number by its factors.

a. **The pure hundreds.**

$$100 = 2 \times 50,\ 4 \times 25,\ 5 \times 20,\ 10 \times 10$$

Therefore,

$$\begin{aligned} 200 &= 2 \times 2 \times 50 = 4 \times 50 \\ & \ 2 \times 4 \times 25 = 8 \times 25 \\ & \ 2 \times 5 \times 20 = 10 \times 20 \\ & \ 2 \times 10 \times 10 = 20 \times 10 \\ 300 &= 3 \times 2 \times 50 = 6 \times 50 \\ & \ 3 \times 4 \times 25 = 12 \times 25 \\ & \ 3 \times 5 \times 20 = 15 \times 20 \\ & \ 3 \times 10 \times 10 = 30 \times 10 = 20 \times 15 \end{aligned}$$

b. **Hundreds with tens.**

$220 = 10 \times 22$. As $10 = 2 \times 5$, $220 = 2 \times 5 \times 22 = 2 \times 110$; and as $22 = 2 \times 11$, $10 \times 2 \times 11 = 10 \times 22$; and as $10 = 5 \times 2$, $5 \times 2 \times 22 = 5 \times 44$.

SIXTH STEP.

Reduction of numbers from 1 *to* 1000 *to their elements.*

It does not matter in what order the numbers are taken, the chief object being practice in rapidly and accurately separating the numbers into their elements. The pupil is now able to tell at a glance into what parts the number must be separated. The teacher should make the work partly oral and partly written.

360.

300 + 60
180 + 180
336 + 24
etc.

3 × 100 + 3 × 20
3 × 120
20 × 18 = 18 × 20
9 × 40 etc.

320 + 45, 2 × 150 + 65, 2 × 182 + 1, 7 × 50 + 15, 14 × 25 + 15, 18 × 20 + 5, etc.

These six steps complete the work of the first quarter. The work now changes from the *pure* to the *applied* number. At this point especial attention should be given to compound numbers, weights, measures, money, etc. This will employ the second quarter. The teacher must supply a great many more examples than are here given, as the "applied number" is of great importance.

B.—All-sided Contemplation of the Applied Number.

(Second Quarter.)

a. The tens.

10 cents = 1 dime.
10 dimes = 1 dollar.
10 dollars = 1 eagle.
3 dimes = 30 cents.
5 dimes = 50 cents or ½ dollar.
50 " = 5 dollars or ½ eagle.
1 dollar = 10 dimes.
2 dollars = 20 "
100 cents = 10 " or 1 dollar.
100 dollars = 1000 dimes.
900 dimes = 30 × 30 dimes.
870 " = 29 × 30 "
840 " = 28 × 30 "
810 " = 27 × 30 "
etc.

Divide the class into two divisions and let the first division give 3 times, and the second division 4 times, the numbers as the teacher names them.

Teacher.	1st Division.	2d Division.
25 dimes.	75 dimes.	100 dimes.
9 "	27 "	36 "

Again, the first division can give 3 times the number given, and the other division add the results.

Teacher.	1st Division.
19 dimes.	5 dollars and 70 cents.
12 "	3 " " 60 "
22 "	6 " " 60 "
2d Division....	15 " " 90 "

In long examples the results of multiplying can be written in order to get the total correctly.

14 dollars 9 dimes.
15 " 7 "
16 " 4 "
17 " 8 "
18 " 6 "
19 " 5 "
102 " 9 "

This will be added as follows, the operations being mental, only the above being set down :

14 dollars 9 dimes + 15 dollars = 29 dollars 9 dimes, + 7 dimes = 30 dollars 6 dimes ; 30 dollars 6 dimes + 16 dollars 4 dimes = 47 dollars, etc.

2. Reverse the process, subtracting 19 dollars 5 dimes from 102 dollars 9 dimes, and so on until the first number, 14 dollars 9 dimes, is reached.

3. Multiply each of these numbers by 2, 3, 4, 5, and add the products, and divide the sum by 2, 3, 4, 5, and see if the sum of the original numbers is obtained.

b. Tens and units.

24 dimes = how many cents ?
24 dozen = " " things ?
24 dollars = " " dimes ?

Pupils must be drilled in tables from

12 × 1 to 24 × 12,
15 × 1 to 15 × 15,
16 × 1 to 16 × 16,
24 × 1 to 24 × 24.

One dollar = 10 dimes = 100 cents.
One half " = 5 " = 50 "
One fourth " = 2½ " = 25 "
One fifth " = 2 " = 20 "
One tenth " = 1 " = 10 "
One twentieth " = ½ " = 5 "
One fiftieth " = ⅕ " = 2 "
One hundredth " = ⅒ " = 1 cent.
100 cents = 10 " = 1 dollar.
50 " = 5 " = ½ "
20 " = 2½ " = ¼ "
etc.

The English weights and measures cannot well be applied to the Grube system, as the scales, instead of being decimal, are varying. And yet, with their present knowledge of the numbers as high as 1000, and with the applications of compound numbers made in the First Course during the first two years, the pupils should be able to master the subject of compound numbers in connection with the above work during the second quarter.

SECOND HALF OF THE YEAR.

The division of the work is as follows:

A.—With abstract numbers.

Written:
1. Numeration.
2. Addition,
3. Subtraction.
4. Multiplication.
5. Division.

B.—With concrete numbers.

Oral:
1. Numeration.
2. Addition.
3. Subtraction.
4. Multiplication.
5. Division.

Mental and written arithmetic must not be separated, as though they were different studies; mental arithmetic is the foundation; here the idea has its beginning; on this the written statement depends.

It becomes necessary in large numbers, and where several numbers are involved, to write them down in figures, so that they can be seen by the eye. The written work may thus be considered as only an assistant to the mental work.

Only the important points of each step will be given, the filling in with material being left to the skill and tact of the teacher.

A.—With Abstract Numbers.

FIRST STEP.

Numeration.

a. (Oral.)

Thousands and millions. If we have the 10 hundreds we have a new unity—the thousand. We can combine thousands into tens of thousands; tens of thousands into hundreds of thousands, etc., exactly as we combined units into tens, tens into hundreds, etc.

1 × 1 unit = 1 unit = 1.
10 × 1 unit = 10 units = 10.
10 × 1 ten = 10 tens = 100.
10 × 1 hundred = 10 hundred = 1 thous. = 1000.
10 × 1 thousand = 10 thousand = 1 ten th. = 10000.
10 × 1 ten thous. = 10 ten. thous. = 1 h'd. th. = 100000.
10 × 1 hund. th. = 10 hund. thous. = 1 million = 1000000.

In the same way:

1 × 2 units = 2 units = 2
10 × 2 " = 20 " = 20
10 × 2 tens = 20 tens = 200
etc.

1 × 3 units = 3 units = 3
10 × 3 " = 30 " = 30
10 × 3 tens = 30 " = 300
etc.

1 unit = 1 × 1 unit
1 ten = 10 × 1 unit = 10 units.
1 hundred = 10 × 1 ten = 100 units.
1 thousand = 10 × 1 hundred = 100 × 1 ten = 1000 units.
etc.

The units are units of the first order, the tens are units of the second order, the hundreds are units of the third order, etc. The units of each order are 10 times more than the units of the preceding order. In each order there can be only 10 units, and the tenth unit is the first of the following order.

b. (Written.)

We will write the number 1852, placing the units, tens, hundreds, and thousands, each in a separate box.

$\boxed{1}$ $\boxed{8}$ $\boxed{5}$ $\boxed{2}$

Counting from the right towards the left, we have units in the first box, tens in the second, hundreds in the third and thousands in the fourth, tens of thousands in the fifth, hundreds of thousands in the sixth, millions in the seventh.

$$\begin{array}{ccccccc} 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ \boxed{6} & \boxed{0} & \boxed{9} & \boxed{2} & \boxed{1} & \boxed{6} & \boxed{0} \end{array}$$

Read: 6 millions.
0 hundred thousands.
9 ten thousands.
2 thousands.
1 hundred.
6 tens.
0 units.

Or,

6 million 92 thousand 1 hundred and 60.

Give the name of the units of the 1st, 3d, 5th, 4th, 6th order. What are units of the 8th order called? etc., etc.

Instead of boxes columns may be used as follows:

9 H. Mil.	8 T. Millions	7 Millions	6 H. Thous.	5 T. Thous.	4 Thous.	3 Hund.	2 Tens.	1 Units.
3	0	5	6	4	8	7	0	0
		2	6	5	9	0	8	4
	7	0	0	9	3	6	4	2
				1	9	8	6	0

The pupils should arrange such columns on their slates, and on the blackboard then write numbers in them from dictation, and then drilled until they know the orders from right to left and from left to right.

Then the numbers may be divided into periods (threes), attention being called to the fact that there are "units," "tens," and "hundreds" of units; also units, tens, and hundreds of thousands, millions, etc.

The periods may be shown as follows:

Million.			Thousand.			Units.		
h.	t.	u.	h.	t.	u.	h.	t.	u.
3	2	5	1	7	6	1	8	0

Then all columns, etc., may be abandoned, and the numbers written in periods. Always have the commas, which separate the periods, placed in at the time of writing the number, and not after all the figures have been written. Thus: 325 million (comma), 176 thousand (comma), 180. Thus the pupil becomes perfectly sure in both writing and reading numbers.

SECOND STEP.

Addition.

ORAL AND WRITTEN.

a. Numbers of one figure:

4 units + 5 units = 9 units (4 + 5 = 9).
4 tens + 5 tens = 9 tens = 90 (40 + 50 = 90).
4 hund. + 5 hund. = 9 hund. = 900 (400 + 500 = 900).
etc.

b. Numbers of two figures and one figure:

43 units + 5 units = 48 units (43 + 5 = 48).
43 tens + 5 tens = 48 tens = 480 (430 + 50 = 480).
etc.

c. Numbers of two figures and two figures:

43 units + 28 units = 71 units (43 + 28 = 71).
43 tens + 28 tens = 71 tens = 710 (430 + 280 = 710).
etc.

Then numbers of three figures and one figure, three figures and two figures, etc., will follow in the same way.

The corresponding method for operations to acquire rapidity. For example:

a. 7 + 8 + 9 + 6
70 + 80 + 90 + 60
700 + 800 + 900 + 600
etc.

b, 25 + 9 + 3 + 8
250 + 90 + 30 + 80
2500 + 900 + 300 + 800
etc.

c. 25 + 36 + 47 + 58
250 + 360 + 470 + 580
2500 + 3600 + 4700 + 5800
etc.

The pupils must be led step by step in written addition as well as oral.

Let them write in columns:

(1)

$$\begin{array}{rl} 36 & \\ 24 & \\ 15 & \\ 23 & \\ 50 & \\ \hline 18 & \text{units} \\ 13 & \text{tens} \\ \hline 148 & \text{units} \end{array} \qquad \text{Shorter:} \quad \begin{array}{r} 36 \\ 24 \\ 15 \\ 23 \\ 50 \\ \hline 148 \end{array}$$

(2)

$$\begin{array}{r} 5946 \\ 847 \\ 239 \\ 6320 \\ \hline 22 \\ 130 \\ 2200 \\ 11000 \\ \hline 13352 \end{array} \qquad \begin{array}{rrrrr} & 5 & 9 & 4 & 6 \\ & & 8 & 4 & 7 \\ & & 2 & 3 & 9 \\ & 6 & 3 & 2 & 0 \\ & & (2) & (1) & (2) \\ \hline 1 & 3 & 3 & 5 & 2 \end{array}$$

Place the number to be "carried" under the column to which it belongs, in parenthesis, using smaller figure.

Pupils should be able to answer promptly questions on the addition processes.

THIRD STEP.

Subtraction.

ORAL AND WRITTEN.

a. Numbers of one figure:

9 units — 5 units = 4 units (9 — 5 = 4).
9 tens — 5 tens = 4 tens (90 — 50 = 40).
9 hund. — 5 hund. = 4 hund. (900 — 500 = 400).
etc.

b. Numbers of two figures with one figure:

12 units — 5 units = 7 units (12 — 5 = 7).
12 tens — 5 tens = 7 tens (120 = 50 = 70).
12 hund. — 5 hund. = 7 hund. (1200 — 500 = 700).
etc.

Continue in the same manner as in addition.

$$a.\quad 4325 - 1123 = \begin{array}{rl} 4325 & \text{Minuend.} \\ 1123 & \text{Substrahend.} \\ \cline{1-1} 3202 & \text{Remainder.} \end{array}$$

b. (Borrowing without ciphers.)

$$(1)\qquad \begin{array}{r} 1\;\overset{16}{7}\cdot\overset{15}{6}\cdot\overset{18}{8}\text{*} \\ 6\;7\;9 \\ \hline 1\;0\;8\;9 \end{array} \quad = \quad \begin{array}{r} 1768 \\ 679 \\ \hline 1089 \end{array}$$

$$(2)\qquad \begin{array}{r} 4\;5\cdot\overset{13}{4}\cdot\overset{12}{2} \\ 4\;1\;5\;9 \\ \hline 3\;8\;3 \end{array} \quad = \quad \begin{array}{r} 4\;5\cdot4\cdot2 \\ 4\;1\;5\;9 \\ \hline 3\;8\;3 \end{array}$$

Where "borrowing" is necessary, make the necessary changes in the minuend, placing smaller figures above to indicate the changes. Then work the same example without making the changes.

d. (Borrowing with ciphers in the minuend.)

$$(1)\qquad \begin{array}{r} 7\;4\cdot\overset{15}{6}\cdot\overset{10}{0} \\ 3\;2\;6\;9 \\ \hline 4\;1\;9\;1 \end{array} \quad = \quad \begin{array}{r} 7\;4\cdot6\cdot0 \\ 3\;2\;6\;9 \\ \hline 4\;1\;9\;1 \end{array}$$

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* Grube placed dots at the bottom where "borrowing" was employed; but in order to avoid confusing them with the later use of the decimal point, we place them at the top. The Germans use the comma for the decimal point, therefore this danger does not arise with them.

The explanation of subtraction is based on addition. Take two numbers:

$$\begin{array}{rrl} a. & 3480 & \\ b. & \underline{2375} & \text{Adding.} \\ c. & 5855 & \end{array}$$

How large is a? ($a = 5855 - 2375$.)
How large is b? ($b = 5855 - 3480$.)
How was c found? (By adding a and b.)
How was a or b found? (By subtracting the known a or b from their sum c.)
What do we call c in the subtraction? (The minuend.)
What do we call the a or b? (The subtrahend.)
What do we seek? (The remainder or difference.)
If the minuend were unknown how would you find it? (By adding the subtrahend and difference.)

$$\begin{array}{r} 5855 \\ -\underline{3480} \\ 2375 \\ +\underline{3480} \\ 5855 \end{array} \qquad \begin{array}{r} 5855 \\ -\underline{2375} \\ 3480 \\ +\underline{2375} \\ 5855 \end{array}\left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \text{Proof.}$$

FOURTH STEP.

Multiplication.

ORAL AND WRITTEN.

1. Multiplier One Figure.

a. Multiplicand One Figure.

3×9 units $= 27$ units $(3 \times 9 = 27)$.
3×9 tens $= 27$ tens $(3 \times 90 = 270)$.
3×9 hund. $= 27$ hund. $(3 \times 900 = 2700)$.
etc.

b. Multiplicand Two Figures.

$$3 \times 29 \text{ units} = 87 \text{ units } \left(3 \times 29 = \begin{array}{r} 3 \times 20 = 60 \\ 3 \times \ \,9 = \underline{27} \\ 87 \end{array}\right).$$

3×29 tens $= 87$ tens $(3 \times 290 = 870)$.
3×29 hund. $= 87$ hund. $(3 \times 2900 = 8700)$.

Proceed in the same manner with multiplicands of three figures, four figures, etc.

2. Multiplier Two Figures.

a. THE PURE TENS.

1. *Multiplicand One Figure.*

60×5 units $= 300$ units ($60 \times 5 = 300$).
60×5 tens $= 300$ tens ($60 \times 50 = 3000$).
60×5 hund. $= 300$ hund. ($60 \times 500 = 30000$).
60×5 thous. $= 300$ thous. ($60 \times 5000 = 300000$).

2. *Multiplicand Two Figures.*

$$60 \times 56 \text{ units} = 3360 \text{ units } \left(\begin{array}{r} 60 \times 50 = 3000 \\ 60 \times 6 = 360 \\ \hline 3360 \end{array}\right).$$

60×56 tens $= 3360$ tens ($60 \times 560 = 33600$).
etc.

Proceed the same as before.

b. MIXED TENS.

$$(1)\ 25 \times 9 \text{ units} = 225 \text{ units } \left(\begin{array}{r} 20 \times 9 = 180 \\ 5 \times 9 = 45 \\ \hline 225 \end{array}\right).$$

25×9 tens $= 225$ tens ($25 \times 90 = 2250$).

$$(2)\ 25 \times 96 \text{ units} = 2400 \text{ units } \left(\begin{array}{r} 20 \times 96 = 1920 \\ 5 \times 96 = 480 \\ \hline 2400 \end{array}\right).$$

25×96 tens $= 2400$ tens ($25 \times 960 = 24000$).

Continue in the same way with a multiplier of three figures, then four, etc., taken with a multiplicand of one figure, then two figures, and so on.

Method for Slate Work.

1. *Multiplier One Figure.*

(1) No carrying:

$$\begin{array}{rr} 3 \times 3213 = 3 \times 3000 = & 9000 \\ 3 \times 200 = & 600 \\ 3 \times 10 = & 30 \\ 3 \times 3 = & 9 \\ \hline & 9639 \end{array}$$

Shorter:

$$\begin{array}{r} 3213\\ 3\\ \hline 9\\ 30\\ 600\\ 9000\\ \hline 9639 \end{array}$$

Shortest:

$$\begin{array}{r} 3213\\ 3\\ \hline 9639 \end{array}$$

(2) Carrying:

$$\begin{array}{rrrrr} 3 \times 3226 = & 3 \times & 3000 = & 9000\\ & 3 \times & 200 = & 600\\ & 3 \times & 20 = & 60\\ & 3 \times & 6 = & 18\\ \cline{4-4} & & & 9678 \end{array}$$

$$\begin{array}{r} 3226\\ 3\\ \hline 18\\ 60\\ 600\\ 9000\\ \hline 9678 \end{array} \qquad \begin{array}{r} 3226\\ 3\\ \hline 9678 \end{array}$$

Explanation of Multiplication.

How many times must I take 112 to get 336?

$$\begin{array}{r} 112\\ 112\\ 112\\ \hline 336 \end{array}$$

How can I express that shorter?

$$3 \times 112 = 336.$$

$$\begin{array}{rl} 112 & a.\\ 3 & b.\\ \hline 336 & c. \end{array}$$

How many numbers have I?
Which number is the entirety? (*c*.)
What is the *a*? (A part of *c*.)

What does the *b* tell us? (How many times we must take the part *a* in order to get *c*.)

What is *c*? (The product.)

How do we get *c* from *a*?

What do we call the number which we multiply in order to get the product?

What is the multiplicand? Multiplier? Product?

The multiplicand and multiplier are together called factors, because they produce the product.

The pupil must be able to explain multiplication somewhat as follows:

209 × 3148 = ?

The multiplicand is 3148. This is a part of the unknown product. The multiplier is 209, and tells me how many times I must take the multiplicand 3148 in order to get the product. I take therefore 3148 209 times. I multiply first by 9 units and get 28,332. I proceed to the tens, and as there are none, pass on to the hundreds. I multiply 3148 by 2, and this product by 100, and this gives me 629600. Add the products, and I get 657,932, the product of 209 × 3148.

FIFTH STEP.

Division. ORAL AND WRITTEN.

A.—Without Remainder.

1. DIVISOR ONE FIGURE. (Quotient unchanged.)

a. Dividend also One Figure.

3 units in 6 units = 2 (6 ÷ 3 = 2).
3 tens in 6 tens = 2 (60 ÷ 30 = 2).
3 hund. in 6 hund. = 2 (600 ÷ 300 = 2).
etc.

b. Dividend Two Figures.

3 units in 18 units = 6 (18 ÷ 3 = 6).
3 tens in 18 tens = 6 (180 ÷ 30 = 6).
3 hund. in 18 hund. = 6 (1800 ÷ 300 = 6).
etc.

c. Dividend Three Figures.

3 units in 186 units = 62 (186 ÷ 3 = 62).
3 tens in 186 tens = 62 (1860 ÷ 30 = 62).
3 hund. in 186 hund. = 62 (18600 ÷ 300 = 62).
etc.

(Quotient increasing according to decimal scale.)

a. One third of 6 units = 2 units (6 ÷ 3 = 2).
One third of 6 tens = 2 tens (60 ÷ 3 = 20).
One third of 6 hund. = 2 hund. (600 ÷ 3 = 200).

b. One third of 18 units = 6 units (18 ÷ 3 = 6).
One third of 18 tens = 6 tens (180 ÷ 3 = 60).

2. DIVISOR TWO FIGURES. (Quotient unchanged.)

a. Dividend also Two Figures.

18 units in 54 units = 3 (54 ÷ 18 = 3).
18 tens in 54 tens = 3 (540 ÷ 180 = 3).
18 hund. in 54 hund. = 3 (5400 ÷ 1800 = 3).

b. Dividend Three Figures.

18 units in 108 units = 6 (108 ÷ 18 = 6).
18 tens in 108 tens = 6 (1080 ÷ 180 = 6).

(Quotient increasing.)

a. $\frac{1}{18}$ of 54 units = 3 units (54 ÷ 18 = 3).
$\frac{1}{18}$ of 54 tens = 3 tens (540 ÷ 18 = 30).
b. $\frac{1}{18}$ of 54 hund. = 3 hund. (5400 ÷ 18 = 300).
etc.

B.—With Remainder.

1. DIVISOR ONE FIGURE. (Quotient also one figure.)

a. Dividend One Figure.

3 units in 7 units = 2 with 1 unit remainder (7 ÷ 3 = 2 [1]).

b. Dividend Two Figures.

3 units in 25 units = 8 with 1 unit remainder (25 ÷ 3 = 8 [1])

The teacher will easily continue this work according to the plan followed in the preceding pages under **A.**

METHOD FOR WRITTEN DIVISION.

A.—Without Remainder.

1. DIVISOR ONE FIGURE.

15936 ÷ 3 = 3 into 15000 + 900 + 30 + 6.

3 into 15000 = 5000
3 " 900 = 300
3 " 30 = 10
3 " 6 = 2
3 " 15936 = 5312

Shorter:

3)15936 the whole.
5312 the part.

One more example will be sufficient to show the method.

$$
\begin{array}{l}
79)42391(536\tfrac{47}{79} \\
\ \ \ \underline{395\cdot\cdot} \\
\ \ \ \ \ 289. \\
\ \ \ \ \ \underline{237.} \\
\ \ \ \ \ \ \ 521 \\
\ \ \ \ \ \ \ \underline{474} \\
\ \ \ \ \ \ \ \ \ 47
\end{array}
$$

In the same manner the teacher will be able to continue these operations, completing all the steps in division. All the steps of these operations are much more fully given in the larger book.

Explanation of Division.

The teacher in explaining division must start with multiplication.

$$
\begin{array}{rl}
1260 & (a) \\
\underline{3} & \times\ (b) \\
3780 & (c)
\end{array}
$$

What is *c*? (The product or whole.)
What is *a*? (The multiplicand, a part of the whole.)
What is *b*? (The multiplier, which tells me how many times the multiplicand is to be taken.)
Find *a* from *b* and *c*.

$$
\begin{array}{l}
(b)\ (c)\ \ (a) \\
3)3780(1260 \\
\ \ \ \underline{3\cdots} \\
\ \ \ \ \ 7\cdot\cdot \\
\ \ \ \ \ \underline{6\cdot\cdot} \\
\ \ \ \ \ 18. \\
\ \ \ \ \ \underline{18.} \\
\ \ \ \ \ \ \ 0
\end{array}
$$

The product or entirety is the dividend; the number that I divide by is the divisor, and the number sought, or result, is the quotient.

c = dividend,
b = divisor,
a = quotient.

What are *a* and *b* with reference to the product or dividend? (Factors.)

The pupil must be able to give an explanation similar to the following:

The number which is contained 3 times in 3780 must be ⅓ of 3780. I find ⅓ of 3780 by dividing it by 3. The divisor 3 is the known factor; the dividend 3780, the product and the quotient is the unknown factor, which will be found when I divide the product by the known factor.

3789 = 37 hund. and 8 tens; ⅓ of 37 hund. = 12 hund. with remainder of 1 hund.; 1 hund. + 8 tens = 18 tens; ⅓ of 18 tens = 6 tens.

Therefore ⅓ of 3780 = 12 hund. + 6 tens = 1260. The quotient 1260 is the number which is contained 3 times in 3780. Therefore 1260 can be taken 3 times from 3780.

CONCRETE NUMBERS.

We now need no especial explanation of the application to concrete numbers either for the teacher or for the pupils. It is recommended that there shall always be oral exercises first, before the blackboard or slate be used, until the pupils are familiar with the expressions.

The teacher should multiply examples embracing all of compound numbers, until the pupil is prompt and accurate in the work. Great attention must be paid to the solution of examples. One example solved understandingly is of more worth than a dozen solved mechanically, according to rule, without understanding the principles involved.*

---

* See complete work on Grube for more examples.

THIRD COURSE.

FRACTIONS.

THE FOURTH YEAR.

In most schools but three years are given to primary work, the fourth being the beginning of the intermediate department. Therefore as this work is designed for primary teachers especially, the subject of fractions need not appear here. We will give, however, Grube's general plan of treating this subject, referring to the larger book for a full treatment of it.

1. As in whole numbers, the unit is the basis upon which all measuring depends, so is it also the basis of fractions. The whole number has been recognized as a part of larger numbers ; so the real fraction will be recognized as a part of the unit. Fractions are divisions of the unit, and may be considered as ***descending lower orders***, while the whole numbers are ***ascending higher orders***.

2. The first step begins with ***the half***, the second with ***the third***, etc.

3. The fraction is treated in a manner analogous to the whole number, as follows:

1. Contemplation of the Pure Number.

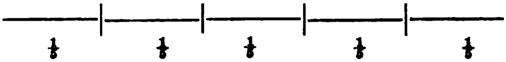
a. Measuring.
b. Comparing.
c. Combining.

2. Application to all the Fundamental Rules.

4. The horizontal line to be divided into parts is taken as the standard of illustration. That it will be ample for the purpose the following illustrations will show.

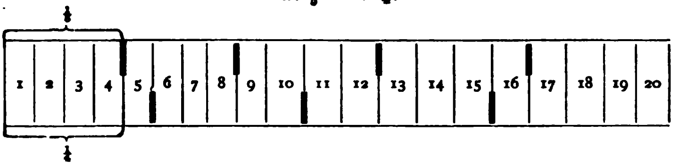
THE FIFTH.

1. *Measuring.*



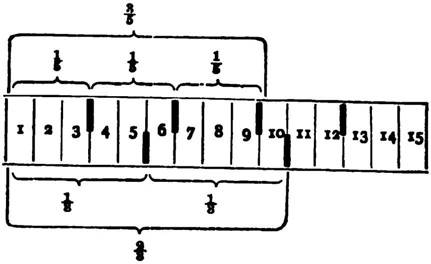
2. *Comparing.*

a. $\frac{1}{5}$ with $\frac{1}{4}$.



The pupil readily sees that $\frac{1}{5}$ and $\frac{1}{4}$ have a common denominator in 20.

b, —$\frac{3}{5}$ with $\frac{2}{3}$



Here the common denominator is seen to be 15. By such illustrations, all the fractions and their relations to each other are taught. Many examples must be given at every step, and applications be made as in whole numbers.

SECOND HALF OF THE YEAR.

The Four Fundamental Rules in Fractions.

The pupils have been thus far taught to consider the fraction from all sides. It remains now to take each operation by itself, and bring the knowledge of it to completion. Let it not be forgotten that accuracy and rapidity must be secured before the pupil can be said to have reached complete mastery of any step.

CLASSIFICATION.

1. Nature and manner of treating the fraction in general (explanation of the parts, kinds, etc., amplifying, common denominator, etc.)
2. Resolution (resolving).
3. Reduction.
4. Addition.
5. Subtraction.
6. Multiplication.
7. Division.

In abstract and concrete numbers oral and written.

There will be no difficulty in connection with these steps, if the preceding have been carefully taught. Grube calls especial attention to the following points:

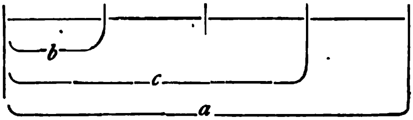
1. *The Unity.*

The pupil has already learned that when 1 is divided into 4 equal parts, one part = ¼.

Three fourths are 3 of the 4 parts, into which 1 has been divided.

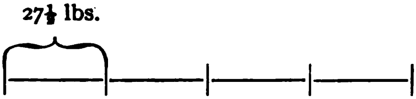
The unity may be of any size or number which is divided into equal parts, as: a yard, a cwt., a ton, etc.

Draw ¼ of a yard, ¾ of a yard.



In order to get b what must I first have? (a.)
What is a? (The whole or unity.)
What quantity in the figure constitutes the unity?
(The yard.)
If we consider a, 1 cwt., as unity, how much does b equal? c? (¼ cwt., ¾ cwt.)
How many pounds would b equal? c?
If b = 25 lbs., how much is the unity or whole?

If *b* = 27¼ lbs., how much is the unity?

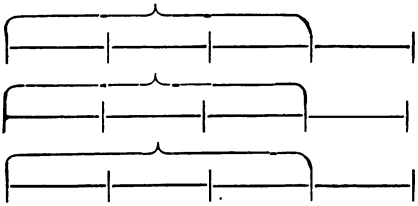


The number with which I indicate the unity is 1.
What do we call the numbers which indicate parts of the unity?
What is ¼? ¾? ⅙? (A fraction.)
What then is a fraction? (A fraction is one or more of the equal parts of a unity.)
How many parts have we in ¼? (We have 1 part.)
In ¾? (We have 3 parts.)
What do we call the 4, or the number below the line?
What do we call the 3, or the number above the line?

The teacher must give many more questions similar to the above, until the pupils understand thoroughly the meaning of the terms *fraction*, *numerator*, and *denominator*.

2. *Expansion and Reduction.*

If you multiply ¾ by 3, what do you get?
Illustrate this.



The method of adding, subtracting, multiplying, and dividing does not essentially differ from that usually given in text-books, and therefore may be omitted here.

This ends the four years' work by Grube's system, and should prepare the child for the average Practical Arithmetic. Two years of work more should be sufficient for the practical duties of life, affording a far better preparation than most children obtain by the usual methods.

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